

TECHNICAL NOTES

The energy balance condition on the interface for phase change with thermal wave effect

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THE INTERFACIAL ENERGY BALANCE CONDITION OF PHASE CHANGE

We consider a control volume V with stationary surface \bar{A} ($= \bar{A}_1 + \bar{A}_2$) as shown in Fig. 1. $A_f(t)$ is the phase-change moving interface between phase 1 and phase 2 that separates V into $V_1(t)$ and $V_2(t)$. \mathbf{n}_k is the unit normal outward from phase k , $k = 1$ or 2, respectively. \mathbf{n}_i is the unit normal to the interface of the phase change, and $\mathbf{n}_i = \mathbf{n}_1 = -\mathbf{n}_2$ is chosen at the interface. \mathbf{v}_i is the phase-change velocity. When \mathbf{v}_i is opposite from \mathbf{n}_1 , a condensing process (gas-liquid) or a freezing process (fluid-solid) happens at the interface.

On the basis of energy conservation law in control volumes $V = V_1(t) + V_2(t)$, the following equation is obtained:

$$\sum_{k=1,2} \frac{d}{dt} \iiint_{V_k(t)} \rho_k \left(c_{ik} T_k + \frac{1}{2} v_k^2 \right) dV$$

$$= \sum_{k=1,2} \left\{ - \iint_{\bar{A}_k} \left[\rho_k \left(c_{ik} T_k + \frac{1}{2} v_k^2 \right) \mathbf{v}_k \cdot \mathbf{n}_k - \mathbf{v}_k \cdot \mathbf{P}_k \cdot \mathbf{n}_k \right] dA \right.$$

$$\left. - \iint_{\bar{A}_k} \mathbf{q}_k \cdot \mathbf{n}_k dA + \iint_{V_k(t)} \mathbf{F}_{bk} \cdot \mathbf{v}_k dV \right\} - \iint_{A_f(t)} \rho_2 Q_L \mathbf{v}_i \cdot \mathbf{n}_i dA$$

where $c_{ik} T_k$ and $\frac{1}{2} v_k^2$ are the internal energy and kinetic energy of phase k , respectively; Q_L is the latent heat of the phase change; while \mathbf{q}_k refers to the heat flux across \bar{A}_k from out-

side; $\iiint_{V_k(t)} \mathbf{F}_{bk} \cdot \mathbf{v}_k dV$ is the applied work by volume force; $\iint_{\bar{A}_k} [\mathbf{v}_k \cdot \mathbf{P}_k \cdot \mathbf{n}_k] dA$ is the applied work by stress at the surface \bar{A}_k ; \mathbf{P}_k ($P_k = -p_k I + \sigma_k$) is the stress tensor of phase k ; \mathbf{v}_k , ρ_k , T_k , p_k , σ_k are the velocity, density, temperature, pressure, and the viscous stress tensor of phase k , and I represents the unit tensor, respectively. The applied work by surface tension at the interface is neglected, and the applied heat source in the control volume is not accounted for.

Employing the Leibniz and Gauss formula, let the integrated functions in the surface integral parts of the above integral be equal to zero, and the energy balance condition at the interface is obtained as follows:

$$\sum_{k=1,2} \left[\dot{m}_k \left(c_{ik} T_k + \frac{1}{2} v_k^2 \right) - \mathbf{v}_k \cdot \mathbf{P}_k \cdot \mathbf{n}_k + \mathbf{n}_k \cdot \mathbf{q}_k \right] + \rho_2 Q_L \mathbf{v}_i \cdot \mathbf{n}_i = 0 \tag{1}$$

where $\dot{m}_k = \rho_k (\mathbf{v}_k - \mathbf{v}_i) \cdot \mathbf{n}_k$ is the relative mass flux of k phase to the interface. For fluid-solid system, equation (1) can be further simplified when the stress and strain in the solid phase are neglected.

THE INTERFACIAL ENERGY BALANCE CONDITION OF THE PHASE CHANGE WITH THERMAL WAVE EFFECT

On the basis of the thermal wave theory, when a temperature gradient is suddenly introduced in an engineering material, the heat flux will be established after a finite relaxation time (τ). Therefore, the relation between the temperature gradient (∇T) and heat flux (\mathbf{q}) can be described as:

$$\mathbf{q}_k + \tau_k \frac{\partial \mathbf{q}_k}{\partial t} + O(\tau_k^2) = -\lambda_k \nabla T_k \tag{2}$$

Noting that enthalpy (sensible heat) is $h_k = c_{pk} T_k = c_{ik} T_k + p_k / \rho_k$, and employing interfacial mass balance condition [8]:

$$\sum_{k=1,2} \dot{m}_k = 0, \text{ then } \sum_{k=1,2} \dot{m}_k c_{ik} T_k$$

$$= \sum_{k=1,2} \dot{m}_k (h_k - h_m) - \sum_{k=1,2} p_k (\mathbf{v}_k - \mathbf{v}_i) \cdot \mathbf{n}_k$$

Where the parameters with subscript m are the parameters at phase-change point. c_{pk} , c_{ik} are specific heat at constant pressure and at constant volume, respectively. Substituting it into equation (1), the following equation is obtained:

$$\sum_{k=1,2} \left[\dot{m}_k (h_k - h_m) + \frac{1}{2} \dot{m}_k v_k^2 + p_k \mathbf{v}_i \cdot \mathbf{n}_k - \mathbf{n}_k \cdot \sigma_k \cdot \mathbf{v}_k + \mathbf{n}_k \cdot \mathbf{q}_k \right]$$

$$+ \rho_2 Q_L \mathbf{v}_i \cdot \mathbf{n}_i = 0 \tag{3}$$

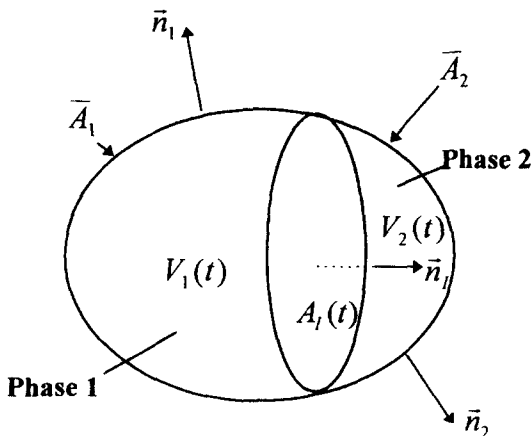


Fig. 1. The control volume $V = V_1 + V_2$ in a two-phase system.

NOMENCLATURE

\bar{A} ($= \bar{A}_1 + \bar{A}_2$)	surface of control volume V
A_I	phase change interface
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_τ	second sonic velocity
e_k^*	strain tensor of phase k
\mathbf{F}_{bk}	volume force of phase k
h	enthalpy
I	unit tensor
L_0	non-dimensional length scale
\mathbf{n}	unit normal vector
Nv	$= v_0^2/h_0$
$N\tau$	$= \tau/(L_0/v_0)$
p	pressure
Pr	Prandtl number
\mathbf{q}	heat flux
Q_L	latent heat of phase change
Re	Reynolds number
St	Stefan number
t	time coordinate
T	temperature
\mathbf{v}	velocity

v_0	non-dimensional velocity scale
Ve	Vernotte number
x_j	space coordinates.

Greek symbols

α	thermal diffusivity
λ	heat conductivity
μ	viscosity
ρ	density
σ	viscous stress tensor
τ	relaxation time of thermal wave.

Subscripts

I	parameter of phase interface
k	parameter of phase k , $k = 1, 2$
m	parameter of phase-change point
0	non-dimensional scale.

Superscripts

'	$\partial/\partial t$
*	dimensionless quantity.

In the following deduction, the heat radiation at the interface is neglected. Noticing that

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x_j} \frac{\partial x_j}{\partial t} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x_j} v_{ij}$$

equation (3) is differentiated with time t ; then substituting equation (2) into it, the terms of $\partial \mathbf{q}_k / \partial t$ are eliminated; assuming $\tau = \tau_1 = \tau_2$, substituting equation (3) into the equation after eliminating the terms of $\partial \mathbf{q}_k / \partial t$, the terms of $\mathbf{q}_k \cdot \mathbf{n}_k$ are further eliminated; after much algebraic operation, we finally obtain:

$$\sum_{k=1,2} \left[\dot{m}_k (h_k - h_m) + \frac{1}{2} \dot{m}_k v_k^2 + p_k \mathbf{v}_k \cdot \mathbf{n}_k - \mathbf{n}_k \cdot \sigma_k \cdot \mathbf{v}_k - \lambda_k \nabla T_k \cdot \mathbf{n}_k \right] + \rho_2 Q_L \mathbf{v}_I \cdot \mathbf{n}_I = -\tau \pi'_0 \cdot \mathbf{n}_I \quad (4)$$

$$\begin{aligned} \pi'_0 &= f \{ \rho_k, \rho'_k, p_k, p'_k, \sigma_k, \sigma'_k, \mathbf{v}_k, \mathbf{v}'_k, \mathbf{v}_I, \mathbf{v}'_I, h_k, h_m, h'_k, Q_L \} \\ &= \sum_{k=1,2} (-1)^{k+1} [\rho'_k (\mathbf{v}_k - \mathbf{v}_I) (h_k - h_m) + \rho_k (\mathbf{v}'_k - \mathbf{v}'_I) (h_k - h_m) \\ &\quad + \rho_k (\mathbf{v}_k - \mathbf{v}_I) h'_k] + (p'_1 - p'_2) \mathbf{v}_I + (p_1 - p_2) \mathbf{v}'_I \\ &\quad - \sum_{k=1,2} (-1)^{k+1} [\sigma_k \mathbf{v}_k + \sigma'_k \mathbf{v}'_k] + \sum_{k=1,2} (-1)^{k+1} \\ &\quad \times \left[\rho'_k (\mathbf{v}_k - \mathbf{v}_I) \frac{v_k^2}{2} + \rho_k (\mathbf{v}'_k - \mathbf{v}'_I) \frac{v_k'^2}{2} + \rho_k (\mathbf{v}_k - \mathbf{v}_I) v_k v'_k \right] \\ &\quad - Q_L (\rho_2 \mathbf{v}_I + \rho_2 \mathbf{v}'_I) \end{aligned}$$

where $G' = (\partial G / \partial t)$. For fluid-solid system, when $P_2 = 0$, $\mathbf{v}_2 = 0$, $P'_2 = 0$, $\mathbf{v}'_2 = 0$, a simplified form of equation (4) can be obtained.

The length, velocity, time, density, pressure and enthalpy are scaled with respect to L_0 , v_0 , L_0/v_0 , ρ_0 , $\rho_0 v_0^2$ and $h_0 (= c_{p0} T_0)$, respectively. Then equation (4) can be transformed into the following dimensionless form:

$$\sum_{k=1,2} \left[m_k^* (h_k^* - h_m^*) + \frac{Nv}{2} \dot{m}_k^* v_k^{*2} + Nvp_k^* \mathbf{v}_I^* \cdot \mathbf{n}_k - \frac{Nv}{Re_k} \mathbf{n}_k \cdot \Xi_k^* \cdot \mathbf{v}_k^* \right]$$

$$- \frac{1}{Pr_k Re_k} \nabla^* T_k^* \cdot \mathbf{n}_k \Big] + St \rho_2^* \mathbf{v}_I^* \cdot \mathbf{n}_I = -\pi'_0 \cdot \mathbf{n}_I \quad (5)$$

$$\pi'_0 = f \{ \rho_k^*, \rho'_k, p_k^*, p'_k, \Xi_k, \Xi_k^*, \mathbf{v}_k^*, \mathbf{v}'_k, \mathbf{v}_I^*, \mathbf{v}'_I, h_k^*, h_m^*, h'_k, Nv, N\tau, St, Re_k \}$$

$$\begin{aligned} &= \sum_{k=1,2} N\tau (-1)^{k+1} [\rho_k^* (\mathbf{v}_k^* - \mathbf{v}_I^*) (h_k^* - h_m^*) \\ &\quad + \rho_k^* (\mathbf{v}_k^* - \mathbf{v}_I^*) (h_k^* - h_m^*) + \rho_k^* (\mathbf{v}_k^* - \mathbf{v}_I^*) h'_k] \\ &\quad + Nv N\tau \{ (\rho'_1 - \rho'_2) \mathbf{v}_I^* + (p'_1 - p'_2) \mathbf{v}'_I \} \\ &\quad - \sum_{k=1,2} \frac{Nv N\tau}{Re_k} (-1)^{k+1} [\Xi_k^* \mathbf{v}_k^* + \Xi_k \mathbf{v}_k] \\ &\quad + \sum_{k=1,2} Nv N\tau (-1)^{k+1} \left[\rho_k^* (\mathbf{v}_k^* - \mathbf{v}_I^*) \frac{v_k^{*2}}{2} \right. \\ &\quad \left. + \rho_k^* (\mathbf{v}_k^* - \mathbf{v}_I^*) \frac{v_k'^2}{2} + \rho_k^* (\mathbf{v}_k^* - \mathbf{v}_I^*) v_k v_k' \right] \\ &\quad - N\tau St (\rho_2^* \mathbf{v}_I^* + \rho_2 \mathbf{v}'_I) \end{aligned}$$

where non-dimensional parameters: $Nv = (v_0^2/h_0)$, $N\tau = (v_0 \tau / L_0)$, Prandtl number of phase k $Pr_k = (c_{p0} \mu_k / \lambda_k)$, Stefan number $St = (Q_L / h_0)$, Reynolds number of phase k $Re_k = \rho_0 v_0 L_0 / \mu_k$, $\Xi_k^* = [\frac{1}{2} (\nabla^* \cdot \mathbf{v}_k^* + 2e_k)]$, and $e_{kij}^* = \frac{1}{2} [(\partial v_{ki} / \partial x_j) + (\partial v_{kj} / \partial x_i)]$ is the strain tensor of phase k . All the quantities with superscript * are dimensionless.

DISCUSSION AND CONCLUSIONS

(1) As shown in equation (4), the enthalpy difference between the two phases, the work of pressure difference imposed on the phase change medium $\sum_{k=1,2} p_k \mathbf{v}_I \cdot \mathbf{n}_k$, the work of viscous stress imposed on fluid $\sum_{k=1,2} \mathbf{n}_k \cdot \sigma_k \cdot \mathbf{v}_k$, the relative kinetic energy of the fluid flow to the moving interface of the phase change $\sum_{k=1,2} \dot{m}_k \frac{1}{2} v_k^2$ must be accounted for in the energy balance condition at the interface of the phase

change, except for familiar latent heat of the phase change and the heat conduction. When the effect of the thermal wave is considered, $\tau \neq 0$, the acceleration of the phase change (\mathbf{v}'_i) and the acceleration of the fluid flow (\mathbf{v}'_k), the compressibility of fluid (ρ'_k), the variation of pressure, viscous stress and enthalpy with time (p'_k, σ'_k, h'_k) must be accounted for further. For actual engineering and scientific problems, equation (4) can be further simplified.

(2) When $\tau \neq 0, p_1 \equiv p_2, \mathbf{v}_k \equiv 0, \rho'_k = 0$, equation (4) is simplified as:

$$\sum_{k=1,2} [\rho_k(c_{pk}T_k - c_{pm}T_m)(-\mathbf{v}_i) \cdot \mathbf{n}_k - \lambda_k \nabla T_k \cdot \mathbf{n}_k] + \rho_2 Q_L \mathbf{v}'_i \cdot \mathbf{n}_I = \tau \pi'_0 \cdot \mathbf{n}_I \quad (6)$$

$$\begin{aligned} \pi'_0 &= f\{\rho_k, \mathbf{v}_i, \mathbf{v}'_i, h_k, h_m, h'_k, Q_L\} \\ &= \sum_{k=1,2} (-1)^{k+1} [\rho_k \mathbf{v}'_i (h_k - h_m) + \rho_k \mathbf{v}_i h'_k] + Q_L \rho_2 \mathbf{v}'_i \end{aligned}$$

The acceleration of the phase change, the enthalpy difference between the two phases and its variation with time must be accounted for in the energy balance condition at the phase change interface, although the pressure difference between the two phases, the fluid velocity and fluid compressibility at the interface are not accounted for. When the control volume is one-dimensional, equation (6) is further simplified as a dimensional form of equation (10) in ref. [7].

(3) When $\tau \neq 0, p_1 \equiv p_2, \mathbf{v}_k \equiv 0, \rho'_k = 0, h_k = h_m, h'_k = 0$, and the control volume is one-dimensional, equation (4) is simplified as equation (2.7) in ref. [3] and equation (11) in ref. [4]. Equation (4) can be simplified as equation (14) in ref. [2] and equation (13) in ref. [1] if $h_k = 0$ instead of $h_2 = 0$.

(4) When $\tau = 0, p_1 = p_2, h_k = h_m, \mathbf{v}_k = 0$, equation (4) reduces to

$$\sum_{k=1,2} \lambda_k \nabla T_k \cdot \mathbf{n}_I = Q_L \rho_2 \mathbf{v}'_i \cdot \mathbf{n}_I \quad (7)$$

This is the classical Fourier relation of phase change. In this relation, the effect of the thermal wave, the fluid velocity, the viscous stress of fluid, the pressure difference and enthalpy difference between two phases at the interface of the phase change are not accounted for.

(5) The non-dimensional parameter $Nv = (v_0^2/h_0)$ is the ratio between the kinetic energy of the physical process and enthalpy; $N\tau = [\tau/(L_0/v_0)]$ is the ratio between the thermal wave relaxation time and the characteristic time of the physical process. When taking v_0 as the characteristic velocity of flow, $N\tau$ is the ratio between the relaxation time and the characteristic time of flow. Nv is the ratio between the kinetic energy of flow and enthalpy. When taking v_0 as the thermal diffusion velocity, i.e. $v_0 = (\alpha/L_0)$, and $\alpha =$ thermal diffu-

sivity, then $N\tau$ is the ratio between the relaxation time and the thermal diffusivity time. Nv is the ratio between the kinetic energy of thermal diffusion and enthalpy. When v_0 is taken as second sonic velocity [$v_0 = \sqrt{(\alpha/\tau)} = C_s$], Nv will be the ratio between the kinetic energy of the thermal wave propagation and enthalpy. $N\tau = (\sqrt{\alpha\tau}/L_0)$ is only just the Vernotte number, Ve , which is the ratio between the relaxation time and characteristic time of the thermal wave propagation ($\tau/(L_0/C_s)$), and is also the square root of the ratio between the relaxation time and the thermal diffusivity time $\sqrt{\tau/(L_0^2/\alpha)}$.

In conclusion, therefore the correct correlation of the experimental data in the phase-change problems would contain the following dimensionless parameters: $f(Re_k, Pr_k, Nv, N\tau, St)$.

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